Universal Taper for Compensation of Step Discontinuities in Microstrip Lines

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Abstract—**A simple closed form expression is derived for the contour of a taper compensating step discontinuities in microstrip lines. The procedure is based on the observation that a function playing a prominent role in the differential equation of the taper admits a simple approximation. The contour formula is shown to be valid, with good accuracy, for a very broad range of conditions.**

I. Introduction

STEPS in the width of microstrip lines are associated with fringing fields resulting in an excess capacitance that has to be either accounted for or compensated by a local change in the shape of the line.

For TEM planar transmission lines, Malherbe [1] proposed a taper, subsequently particularized by Hoefer [2] for microstrip lines. In the coordinate system xO_1y shown in Fig. 1, Hoefer's contour is described by the equations:

$$\frac{dy}{dx} = \sqrt{\left\{\frac{\dfrac{\eta_0 h}{2y\epsilon_r} \cdot \dfrac{\epsilon_e(w_2)}{Z_{01}(w_2)} - 1}{\dfrac{\eta_0 h}{2y\epsilon_r} \dfrac{\epsilon_e(2y)}{Z_{01}(2y)} - 1}\right\}^2 - 1}\,, \quad \text{(1a)}$$

for the symmetric case, and

$$\frac{dy}{dx} = \sqrt{\left\{\frac{\dfrac{\eta_0 h}{y\epsilon_r}\left[\dfrac{2\epsilon_e(w_2)}{Z_{01}(w_2)} - \dfrac{\epsilon_e(y)}{Z_{01}(y)}\right] - 1}{\dfrac{\eta_0 h}{y\epsilon_r} \dfrac{\epsilon_e(y)}{Z_{01}(y)} - 1}\right\}^2 - 1}\,, \quad \text{(1b)}$$

for the nonsymmetric case.

In these relations, ϵ_e is the effective dielectric constant of a constant width microstrip line on a substrate of dielectric constant ϵ_r and Z_{01} is the characteristic impedance of a similar line, air-filled. Their expressions, given in [2], are so complicated, that integrating analytically equation (1) seems hopeless; therefore Hoefer suggests a numeric method that approximates the contour by a sequence of small steps. On this basis, Hutchings et al. [3] developed a computer program that generates the required contour.

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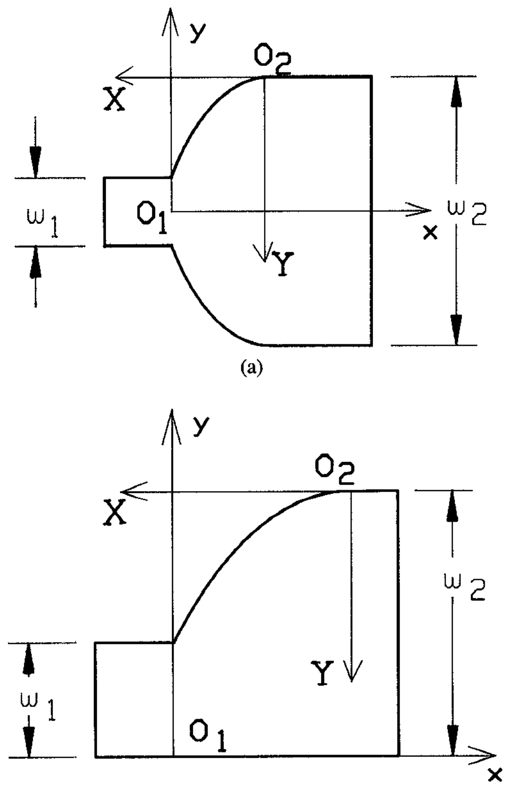

Fig. 1. Change in width of a microstrip line, compensated for parasitics. (a) Symmetric case. (b) Nonsymmetric case. Coordinate system (xO_1y) is used in (1a), (1b). The coordinate system (XO_2Y) is used in (7).

II. Derivation of the Contour Equation

An examination of the relations (1a), (1b) reveals that in both intervenes the function

$$F(u) = \frac{\eta_0}{\epsilon_r} \frac{\epsilon_e(u)}{Z_{01}(u)}\,, \quad (2)$$

where u is the line width/height ratio ($2y/h$ or y/h, respectively). Introducing also a normalized longitudinal variable ($v = x/h$), (1a) can be written as:

$$\frac{du}{dv} = 2\sqrt{\left[\frac{F(u_M) - u}{F(u) - u}\right]^2 - 1} \quad \text{(3a)}$$

and (1b) as:

$$\frac{du}{dv} = \sqrt{\left[\frac{2F(u_M) - F(u) - u}{F(u) - u}\right]^2 - 1}\,, \quad \text{(3b)}$$

where we used the notation $u_M = w_2/h$ (see Fig. 1).

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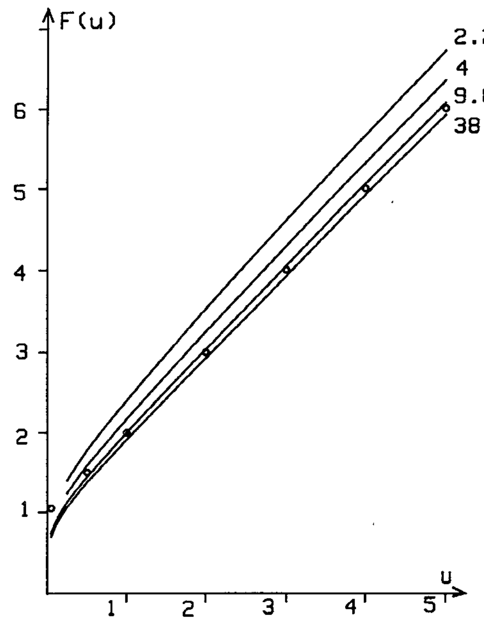

Fig. 2. Plot of $F(u)$ for several frequently used values of ϵ_r. (Small circles indicate values of $\Phi(u) = u + 1$).

Fig. 2 shows a plot of $F(u)$ for various ϵ_r. With increasing ϵ_r the curves converge, approaching a limit curve very close to a straight line, $\Phi(u) = u + 1$ (indicated in Fig. 2 only by a few discrete points).

If we replace $F(u)$ by $\Phi(u)$, (3a), (3b) become respectively:

$$\frac{du}{dv} = 2\sqrt{(u_M - u)^2 + 2(u_M - u)} \tag{4a}$$

and

$$\frac{du}{dv} = \sqrt{(u_M - u)^2 + (u_M - u)}\,. \tag{4b}$$

Both these equations can now be integrated easily, yielding:

$$u_M - u = 2\sinh^2(v_M - v) \tag{5a}$$

and, respectively,

$$u_M - u = \sinh^2(v_M - v), \tag{5b}$$

where $v_M = L/h$ and L is the axial length of the taper.

By defining a new system of coordinates XO_2Y (Fig. 1), where in the symmetric case

$$Y = \frac{1}{h}\left(\frac{w_2}{2} - y\right) \tag{6a}$$

and in the nonsymmetric case

$$Y = \frac{1}{h}(w_2 - y), \tag{6b}$$

while in both cases

$$X = \frac{1}{h}(L - x), \tag{6c}$$

(8a) and (8b) are seen to be reduced to:

$$Y = \sinh^2 X \tag{7}$$

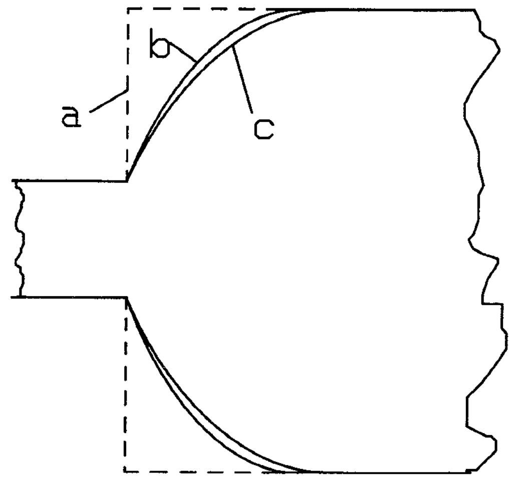

Fig. 3. Change in width of a microstrip line on duroid ($\epsilon_r = 2.2$). (a) Uncompensated step. (b) Taper according to (7). (c) Optimum contour, obtained by numeric integration of (1a).

This simple result has a high degree of generality, since it applies for a step of any size, for any line widths or substrate thickness, independently on ϵ_r (provided it is large enough).

III. Accuracy of the Contour Equation

In most practical cases, the contour described by (7) is hardly distinguishable from the optimum contour, generated according to Hoefer's equations (1a) or (1b).

Although the approximation of $F(u)$ by $\Phi(u)$ deteriorates for $u \ll 1$, the formula (7) can be shown to closely approximate the optimum taper even for open-circuited stubs (when $w_1 \to 0$).

Regarding the applicability of (7) for substrates with lower dielectric constant, an indication can be given by the example illustrated in Fig. 3, for a relatively large symmetric step (from $w_1 = h$ to $w_2 = 4h$ in a microstrip line on duroid ($\epsilon_r = 2.2$). The figure shows the optimum contour c, —as given by the Hoefer's equation (1a)—and the contour b, given by (7). It can be seen, by comparing the two contours with the uncompensated step in Fig. 1(a), (drawn in dashed line), that even in this rather extreme case formula (7) leads to a fairly close approximation, the slight undercompensation of the parasitics remains perfectly acceptable for most applications. For increasing ϵ_r, the optimum contours will converge fast to the curve given by (7). It seems thus safe to extend the applicability of (7) for all the substrates used in practice.

Ultimate accuracy can be obtained, for substrates of low dielectric constant, by a more general closed-form expression for the taper, the derivation of which is given in Appendix. It still is more convenient than the numeric integration of (1a), (1b), but even so, the loss in simplicity and generality as compared to (7) is justifiable only in the most extreme circumstances.

IV. Conclusion

A simple formula has been derived for the contour which compensates the step discontinuity in a microstrip line. It is easy to implement and has a very broad validity. It should be noticed that the reference plane (where the step in impedance is required) is not the origin of the system of coordinates (O_2) but rather the section where the contour intersects the narrower line (see Fig. 1(a)).

Appendix

For smaller values of ϵ_r, Fig. 2 indicates that $F(u)$ remains almost linear, so that an approximation by $\Psi(u) = Au + B$ is applicable. Replacing $F(u)$ by $\Psi(u)$ in (3), the equation can still be analytically integrated, and the result is:

$$X = \frac{1}{\sqrt{A(2-A)}} \cdot \left[\sinh^{-1}(\sqrt{Y}) - (A-1)\sqrt{Y(1+Y)}\right], \quad (8)$$

where X and Y (Fig. 1) represent the geometric coordinates normalized to

$$h_{\text{eq}} = \frac{Bh + (A-1)w_2}{2-A}. \quad (9)$$

Equation (8) however depends on the nature of the substrate through A and B and on the width of the lower impedance line, w_2. Considering also the sacrifice in simplicity as compared to (7), the price seems high for a refinement which (as suggested by Fig. 3) would rarely be necessary.

References

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